

MODELING NON-GRAVITATIONAL FORCES ACTING ON TOPEX/POSEIDON: THE EARLY DAYS

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TOPEX/POSEIDON is a satellite mission that will use altimetry to make precise measurements of sea-level. The principal goal is to measure sea-level with unprecedented accuracy such that small-amplitude, basin wide sea-level changes caused by large-scale ocean circulation can be detected. To **reach** this goal, the sensor system and orbit must measure sea-level with decimeter **accuracy**. This requires that the radial component of the orbit be known to the decimeter level. Orbital errors are dominated by **mismodelled gravitational** and non-gravitational forces. This paper presents our analysis of non-gravitational forces acting on the satellite during the early days of the mission. Studies were conducted by comparing direct estimates of these forces with observed perturbations in the mean orbital elements. The **results** show that the satellite is experiencing an unexpected **along-track** acceleration. **Hypotheses** range from out-gassing to thruster leaks and drag forces to radiative force. Currently, these issues have not **been** resolved; however, the evidence suggests that out-gassing was dominant during the **first** weeks of the mission and that **thermal** imbalances **persist**.

MISSION OVERVIEW

TOPEX/POSEIDON (T/P) is a satellite mission that will use altimetry to make precise measurements of sea level; the primary goal is the study of global ocean circulation. The mission is jointly conducted by the United States National Aeronautics and Space Administration (NASA) and the French space agency, **Centre National d'Etudes Spatiales (CNES)**. The launch of the satellite took place on August 10, 1992. The primary mission will last for 3 years, with the possibility of an extended mission for an additional 2 years. The principal goal of T/P is to measure sea level with unprecedented accuracy such that small-amplitude, basin wide sea-level changes caused by large-scale ocean circulation can be detected. To reach this goal, the sensor system and orbit must measure sea-level with decimeter accuracy.

The orbit enters into the measurement of sea level in two important ways. It dictates the temporal and spatial sampling pattern of the altimeter, which in turn dictates our ability to measure certain sea-surface features; therefore, our knowledge of the radial component of the orbit, as obtained through the process of orbit determination, is of great

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importance. **There** are two essential elements required for **precision** orbit determination, the tracking system and the dynamical models.

To support orbit determination, the T/P satellite is configured with a Laser **Retroreflector** Array (**LRA**) (NASA), a Doppler **Orbitography** and Radio-positioning Integrated by Satellite (DORIS) Dual-Doppler Tracking System Receiver (**CNES**), and a Global Positioning System Demonstration Receiver (**GPSDR**) (NASA), which is experimental. The LRA will be used with a network of 10 to 15 satellite laser ranging (**SLR**) stations to provide the NASA baseline tracking data for precision orbit determination. The DORIS tracking system will provide the CNES baseline tracking data using **microwave** Doppler techniques for precision orbit determination. The DORIS system has been successfully demonstrated by the SPOT-2 Mission. The system is composed of an onboard receiver and a network of 40 to 50 ground transmitting stations, providing **all-weather**, global tracking of the satellite. The signals are transmitted at two frequencies (401.25 MHz and 2036.25 MHz) to allow removal of the effects of ionospheric free electrons in the tracking data. The GPSDR, operating at 1227.6 MHz and 1575.4 MHz, will use a new technique of GPS differential ranging for precise, continuous tracking of the spacecraft with better than decimeter accuracy. **Only** SLR measurements will be used in the current analysis.

Since tracking provided by SLR is not a direct measurement of the orbital state, and is generally not continuous in time, dynamical equations are required to produce a continuous, precise orbit for the mission. To achieve the expected 13-cm (global rms) radial orbit accuracy for the mission, a significant amount of effort has been expended to reduce errors due to Earth gravity field and non-gravitational **force** models. To maximize the accuracy of orbit determination, a high-altitude orbit is preferred because of the reduced atmospheric drag and the reduced gravity perturbations acting on the satellite. The height of the orbit is limited by the increased power needed by the altimeter to achieve the required signal-to-noise ratio. A compromise is in the range of 1200 to 1400 km. Within this range, the exact altitude that **allows** the orbit to satisfy all other constraints, e.g. it must fly over the two verification sites (Point Conception off California and **Lampedusa** Island in the Mediterranean Sea) is 1336 km. Shown in Table 1 are the characteristics of the baseline mission orbit.

Table 1. Characteristics of the Operational Orbit,

Mean elements

Semimajor axis, km	$a = 7714.4278$
Eccentricity	$e = 0.000095$
Inclination, deg	$i = 66.039$
Inertial longitude of ascending node, deg	$N = 116.5574$
Argument of perigee, deg	$w = 90.0$

Auxiliary data

Reference equatorial altitude, km	$h = 1336$
Nodal period, s	$P = 6745.72$
Cycle (127 revs) period, days	$T = 9.9156$
Inertial nodal rate, deg/day	$W = -2.0791$
Longitude of equator crossing of pass 1, deg	$I = 99.9242$
Acute angle of equator crossing, deg	$x = 39.5$
Ground-track velocity, km/s	$v = 5.8$

Table 2. Gravitational Force Model Summary	
Central Body Perturbations:	
	JGM-1 geopotential model;
	$GM_e = 398600.4415 \text{ km}^3\text{s}^{-2}$; $a_e = 6378136 \text{ m}$.
Third Body Perturbations:	
	Point mass Sun, Moon and planets with DE-200 ephemeris [Standish, 1982].
Solid Earth Tides Perturbations:	
	Frequency dependent k_2 using Wahr formulation [Wahr, 1981];
	Frequency independent Love number (k_2) = 0.3, Lag angle (δ) = 0;
	Permanent tide correction to \bar{C}_{20} applied (ACM = -1.391 E-8 per unit k_2).
Ocean Tides Perturbations:	
	Merit ocean tide model [Melbourne <i>et al.</i> , 1983; Eanes <i>et al.</i> , 1982].
General Relativity Perturbations:	
	One-body relativistic perturbations [Moyer, 1971].

FORCE MODELS FOR ORBIT DETERMINATION

Table 2 gives a summary of gravitational models used for this study while the focus of the current analysis is on non-gravitational forces. At an altitude of 1336 km, solar radiation is the largest non-conservative force acting on the spacecraft. In addition to direct solar radiation from the Sun, the Earth's **albedo** and infrared (IR) emissions, along with the **infrared** (thermal) emissions from the spacecraft itself, will perturb the orbital motion. The time rate of change of solar energy per unit area incident on a body at one astronomical unit (AU) is approximately 1368 W/m^2 [Kivelson, 1986]. It varies by 41 W/m^2 over a solar year due to the eccentricity of the Earth's orbit. The Earth's **albedo** contributes an average of 465 W/m^2 and the Earth's IR radiation contributes an average of 232 W/m^2 for a satellite at the altitude of T/P [Knocke *et al.*, 1988]. The spacecraft itself radiates thermal energy according to the Stefan-Boltzmann law ($5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$), where it is the thermal imbalance, i.e. the differential radiation across the body of the spacecraft, that is of importance.

The total reflective (solar radiation pressure) and thermal forces acting on T/P area complex function of true anomaly and the angle between the earth-sun line and orbit plane. This is because the yaw of the satellite must continually change to ensure that the front of the solar array is exposed to the sun. At the same time, the boresight of the altimeter must always remain pointed at nadir. Over a solar year, the RMS of force due to solar radiation pressure is approximately 46 kilowatts and the thermal forces on the order of 3.3 kilowatts. Earth IR contributes 4.2 kilowatts in the radial direction. The solar radiation pressure is clearly the dominant force. The acceleration can be found by dividing the force by the speed of light and the mass of the spacecraft (approximately 2400 kg) which results in $4/3 \text{ nm/s}^2/\text{kW}$, e.g. the forces given above are equivalent to accelerations of 61, 4.4, and 5.6 nm/s^2 respectively. A thorough discussion of the radiation forces acting on T/P has been given by Marshall *et al.*.

The drag forces acting on T/P result in an acceleration on the order of 0.3 nm/s^2 . This is at least two orders of magnitude smaller than solar radiation pressure and at least one order of magnitude less than the thermal forces. As will be discussed below, even accelerations as small as those due to drag can be detected using T/P precision orbit determination techniques.

An empirical acceleration model is also used for investigating the non-gravitational forces acting on the TOPEX/POSEIDON. The mathematical formulation is 'given as,

$$\begin{aligned}\ddot{\mathbf{r}}_{\bullet \text{ empirical}} = & P_r + \sum_{i=1}^2 (C_r^i \cos iu + S_r^i \sin iu) \\ & + P_t + \sum_{i=1}^2 (C_t^i \cos iu + S_t^i \sin iu) \\ & + P_n + \sum_{i=1}^2 (C_n^i \cos iu + S_n^i \sin iu)\end{aligned}$$

where

u = argument of latitude of the satellite,
 P_r, P_t, P_n = constant acceleration in radial, **transverse**, and normal directions,
 C^i, S^i = coefficients for acceleration with a **frequency** of i cycles/revolution,

Studies shown that this empirical acceleration model is very effective for modeling **non**-gravitational forces acting on an Earth orbiting spacecraft. The following analytic and orbit determination results are based on these empirical perturbation functions,

PERTURBATION THEORY

Small forces acting on T/P can not be observed directly, so we are disposed to inferences afforded by the process of orbit determination. We begin with a brief review of the perturbation equations for **Keplerian** orbit elements [McCuskey, 1963]. Applying the small eccentricity approximation, which is appropriate for TOPEX/POSEIDON, it is found that

$$da/dt = 2n^{-1}S$$

$$de/dt = (na)^{-1}[\sin f]R + \cos f]S]$$

$$ds/dt = [(nae)^{-1}\sin f] - 2(na)^{-1}]R - 2(nae)^{-1}\sin f]W$$

$$dN/dt = (na)^{-1}[\sin(u)/\sin(i)]W \quad dN/dt = (na)^{-1}[\sin(u)/\sin(i)]W$$

$$dw/dt = -(nae)^{-1}\cos f]R + 2(nae)^{-1}\sin f]S - (na)^{-1}\sin(u)\cot(i)W$$

$$\frac{di}{dt} = (n a)^{-1} \cos(u) W$$

where

a	= semi-major axis
e	= eccentricity
i	= inclination
w	= argument of periapsis
N	= longitude of the ascending node
s	= $-nT$, i.e. the product of mean motion and time of periapsis passage; mean anomaly
f	= true anomaly
u	= argument of latitude = $w + f$
n	= <i>the</i> mean motion

The orthogonal components of the acceleration acting on the spacecraft are R , S , and W which are radial, transverse (along-track), and normal components respectively. A rigorous analysis of these equations would require the application of **general** perturbation techniques, however rough order of magnitude estimates can be obtained by assuming that, over the period of interest, a , e , i , and w are constant when used on the right-hand side of these equations.

With these expressions, we can approximate the expected variations in the mean orbital elements due to small forces acting on the satellite. For example, perturbations in semi-major axis (a) are affected only by an along track component of force. If the force is constant, a will vary secularly with time. Therefore, variations in a , particularly secular ones, provide a direct measure of an along-track force. Such a force would interact directly with the drag force and, in some cases, possibly be mistaken for drag.

Perturbations in eccentricity (e) are affected by both radial and transverse forces and are proportional to $\sin(f)$ and $\cos(f)$ respectively. If the forces are constant, e will vary sinusoidally at the orbital period, in which case, the change in the mean eccentricity over one orbit would be zero. However, if the forces act over only a portion of the orbit, e will undergo secular variations. For example, a constant transverse or radial force resulting from solar radiation pressure can not be detected by observing variations in e , except when the satellite is being occulted.

The normalized time of **periapsis** passage (s) undergoes large perturbations in the presence of small radial and transverse forces due to the singular nature of **Keplerian** elements. In addition, s will undergo secular variation in the presence of a constant radial force. Argument of **periapsis** passage (w) varies in a manner similar to time of **periapsis** passage, again due to the singular nature of **Keplerian** elements. In addition, w will vary at the orbital period in the presence of a constant normal force. Due to the large erratic variations in s and w , it is difficult to use them to obtain a measure of perturbing forces.

Perturbations in the longitude of ascending node (N) are affected only by a normal component of force. If the force is constant, N will vary secularly with time. Inclination (i) depends only on a normal force as well. If the force is constant, i will vary sinusoidally at the orbital period. In such a case, the change in mean inclination over one orbit would be zero. However, if the force acts over only a portion of the orbit, i will undergo secular variations. Clearly then, N is the best indicator of a normal force, however, similar to e , i

can be used to observe a constant normal force due to radiation pressure during occultation season.

ORBIT DETERMINATION RESULTS

Precision orbits were determined by processing SLR data while solving for the spacecraft state, daily constant along-track accelerations, and daily once-per-revolution accelerations in the along-track and orbit-normal directions. Data batches of 1-day arcs were used for the early 50 days of mission and 10-day arcs for the first 12 10-day cycles. Some daily arcs were not fit during the early days because of interrupts in the orbit due to maneuvers or bad SLR tracking.

Estimates obtained for the accelerations during the early days of **TOPEX/POSEIDON** are shown in Figure 1. The figure also shows a time history of the drag coefficients that would be necessary to replicate the along-track forces. Based on pre-launch analysis, it was expected that the accelerations would be less than 1 nm/s^2 , 50 nm/s^2 , and 30 nm/s^2 for the constant along-track, once-per-revolution along-track, and once-per-revolution normal components respectively. Values for the drag coefficients were expected to be on the order of 2. Examination of the acceleration and drag coefficients show that unexpectedly large transverse forces were acting on the satellite during the early days of the mission, particularly during the first two to three weeks. This is most evident in the constant along-track acceleration component, which is normally dominated by drag. Perturbation theory shows that a constant along-track acceleration will introduce secular variations in semi-major axis and periodic variations in eccentricity and argument of **periapsis**. A -1 nm/s^2 along-track acceleration will cause the **TOPEX/POSEIDON** orbit to decay at a rate of approximately -19 cm/day . During the early days of the mission, drag accelerations were expected to be on the order of $0.3\text{--}0.5 \text{ nm/s}^2$. Figure 2 shows the time history for the semi-major axis derived from the orbits determined using the approach described above. Clearly, the forces experienced by the satellite were not due to drag.

It is important to the success of the mission to be able to model these forces, not only for precision orbit determination, but also for navigation. The navigation team has a requirement to maintain the ground-track of **TOPEX/POSEIDON** to within 1 Km, which requires good a priori knowledge of orbit decay. Figures 3 and 4 show a continuation of this analysis up to the present time. Between days 270-318, the **along-track** force remained relatively small and at times introduced subtle growths in semi-major axis. On day 318, the orientation of the satellite was fixed at a yaw angle **normal** to the mean yaw angle maintained up until that time. (Yaw maneuvers are performed twice each solar year to properly illuminate the solar array. For periods other than fixed yaw, the satellite oscillates about a mean yaw angle where the amplitude of the oscillation is largest on each side of the fixed yaw regime. This process is called yaw-steering). For the next six days, at which time another yaw maneuver was performed to 'flip' the satellite through 180° of yaw, the magnitude of the anomalous along-track force increased substantially. This was of course accompanied by a rapid decay in the orbit. The yaw then remained fixed until day 329, at which time the satellite returned to slew about a yaw angle 180° out of phase with the mean yaw maintained during the previous slew period. In other words, the opposite side of the satellite was being presented to the drag force. Up until the next fixed yaw sequence, which began on day 364, the along-track force appears to be close to normal drag.

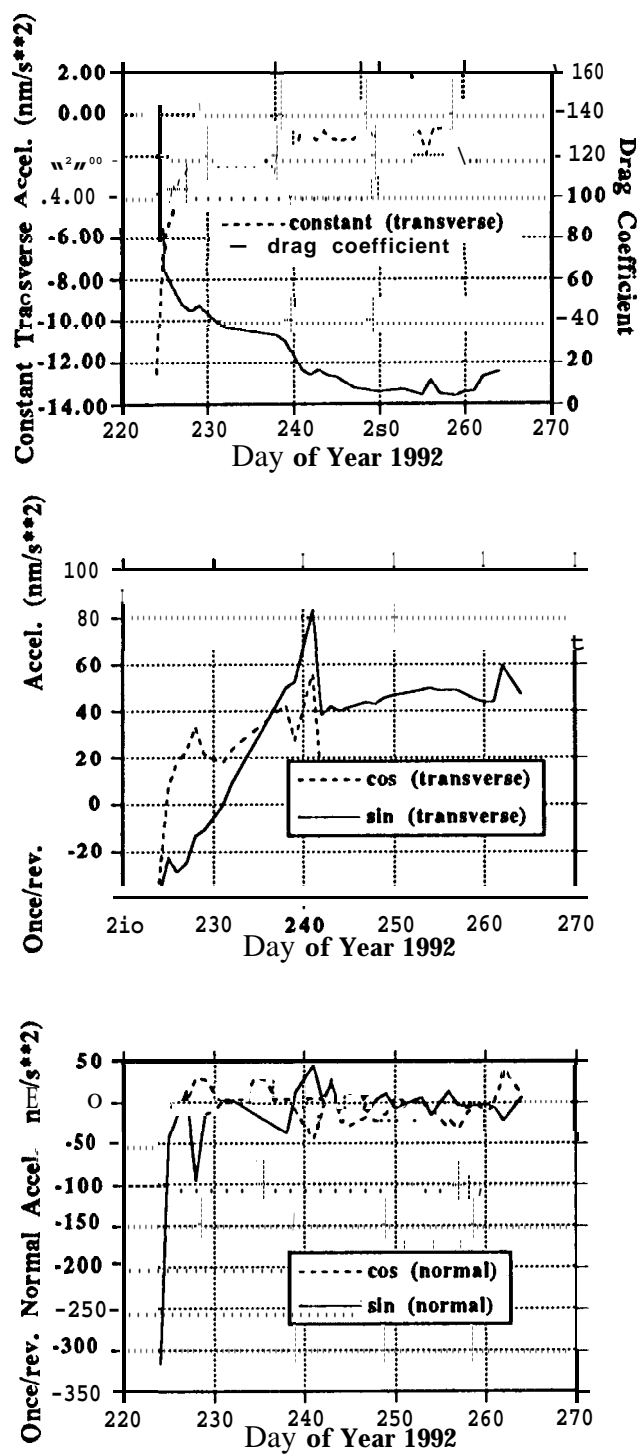


Figure 1. Estimated empirical accelerations for TOPEX/POSEIDON: the early days

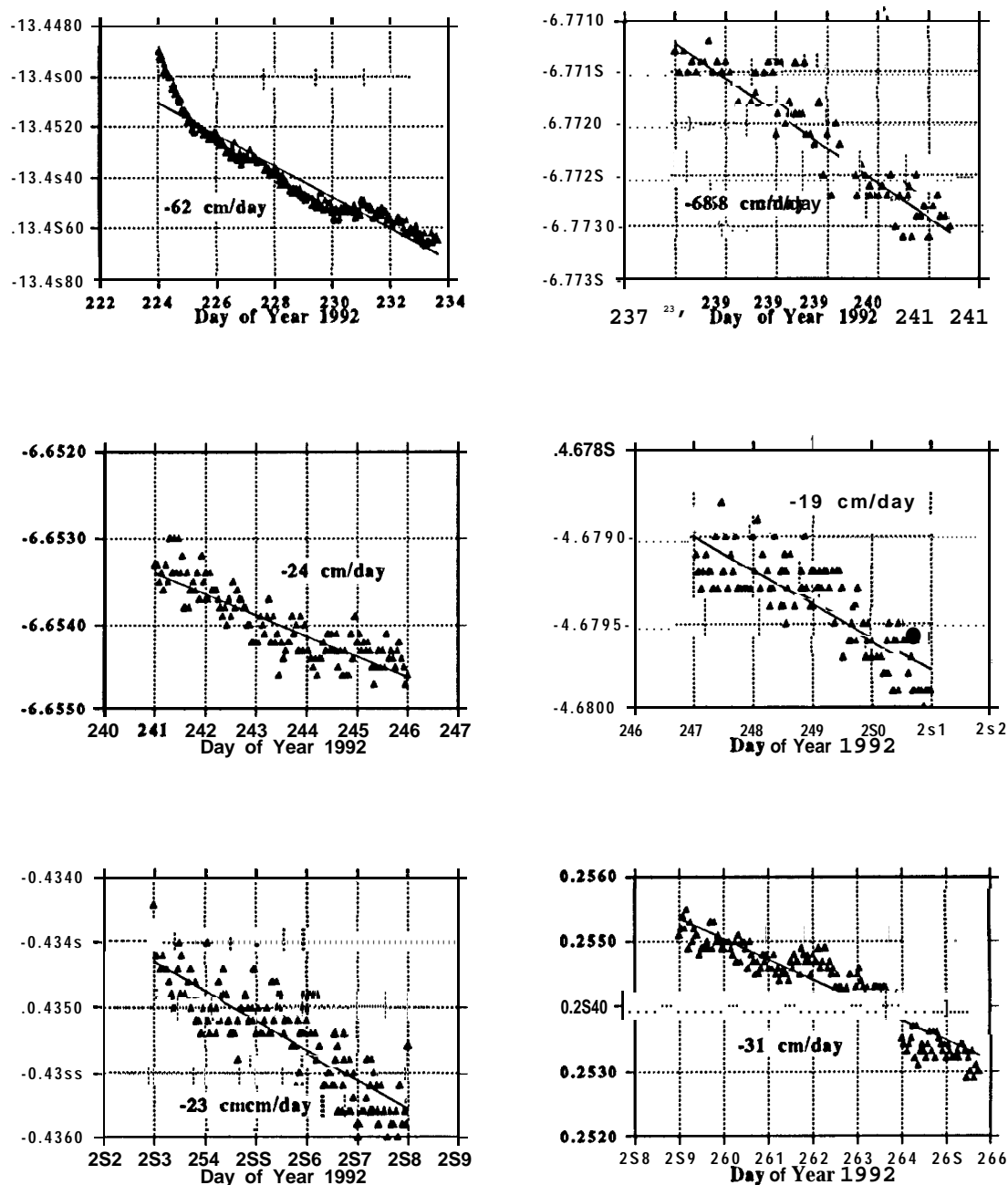


Figure 2. TOPEX/POSEIDON mean semi-major axis
(subtracted from 7714 km) :the early days

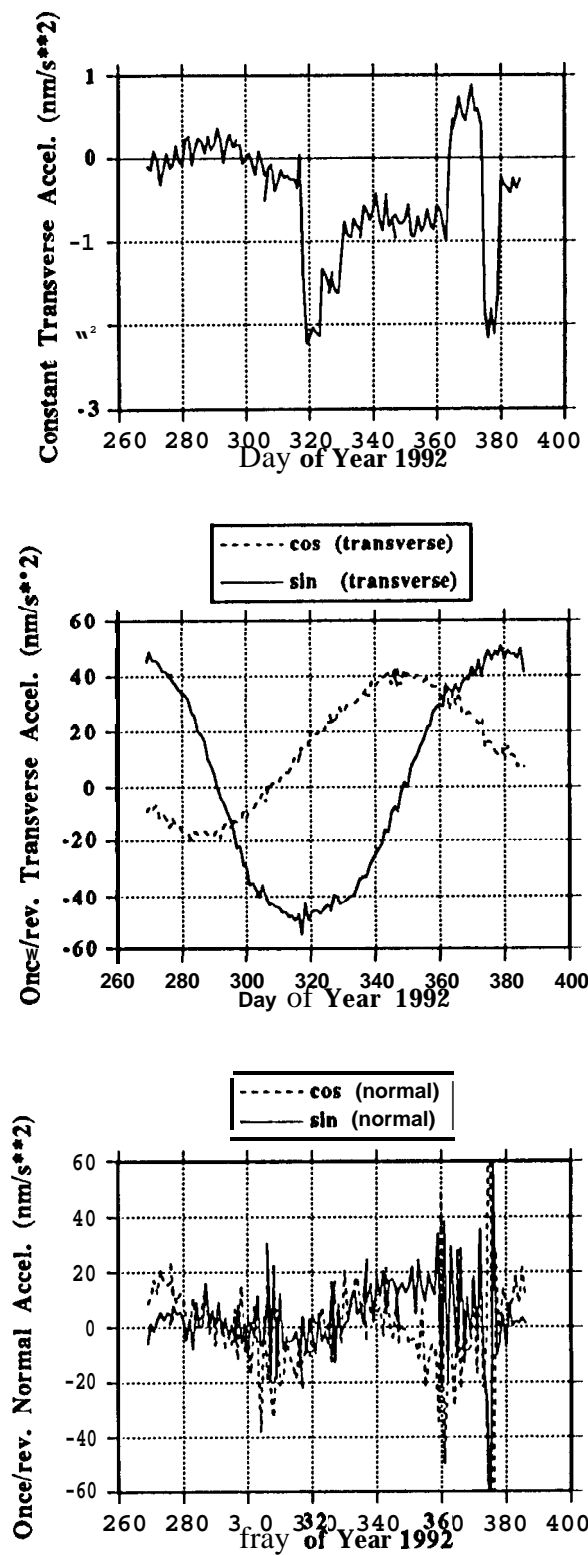


Figure 3. Estimated empirical accelerations for TOPEX/POSEIDON: the 10-day cycles

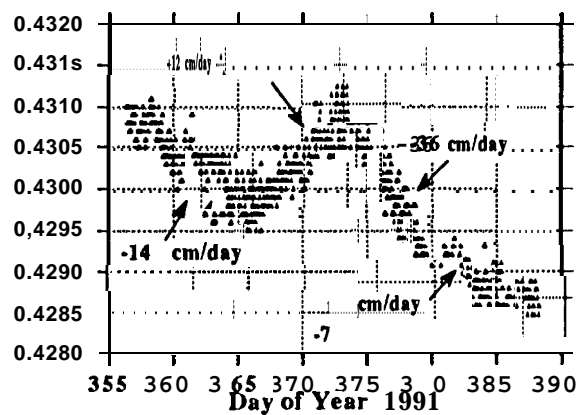
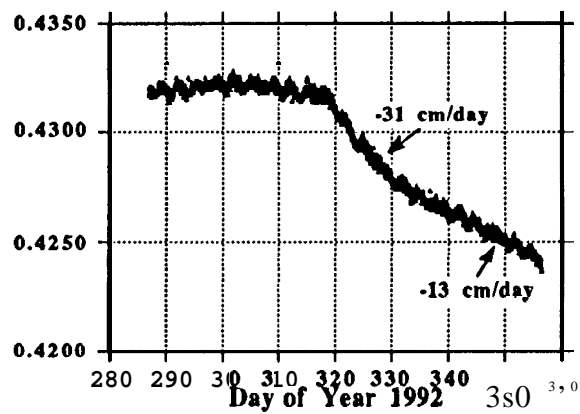
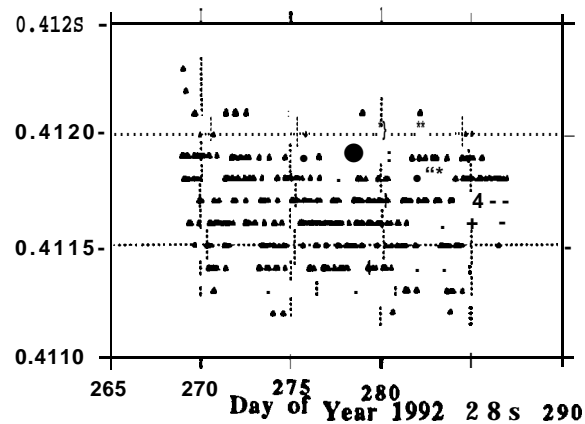


Figure 4. TOPEX/POSEIDON mean semi-major axis (subtracted from 7714 km): the 10-day cycles

In terms of spacecraft orientation, the fixed yaw sequence that took place shortly after the first of the year was basically a mirror image of the previous fixed yaw sequence. The anomalous along-track force behaved according] y. In fact, the semi-major axis actually grew for the first ten days of 1993. We note that the behavior of the anomalous force is dictated by the orientation of the satellite relative to the drag vector. The analysis is complicated by the fact that the solar array pitch angle was also being maneuvered throughout most of the mission to extend the life of the storage batteries. A detailed analysis of the anomalous along-track force continues.

The amplitudes of the estimated along-track and normal once-per-revolution accelerations are also shown in Figure 3. The amplitudes of the along-track component vary with the period of a solar year and are approximately 45° out of phase. The normal components are much noisier but appear to vary with a period of half a solar year. Perturbations resulting from these forces **will** be present in all of the orbital orientation parameters and are thereby more difficult to assess than drag-like forces.

COMMENTS

Referring to the perturbation equations given above, the semi-major axis of the orbit will decay at a rate of $19 \text{ cm/day per nm/s}^2$ of along-track acceleration. We would thus expect to see a 5.7 cm/day decay in semi-major axis due to drag. Examination of the orbits determined during the first few days of the mission indicate a decay of approximately $1\text{-}2 \text{ m/day}$. The decay settles down to approximately 60 cm/day after about a week, and it continues to fall off to approximately 25 cm/day a month later. After six to seven weeks into the mission the acceleration starts to look drag-like, however, it soon goes slightly positive, i.e. the semi-major axis actually begins to grow. These results show that the satellite is experiencing non-gravitation forces much larger than drag. Perturbation theory can be used to demonstrate that an along-track force of this size is not due to solar radiation pressure, so the only reasonable explanation is thrusting or thermal imbalance. During the first few weeks of the mission, there is little doubt that satellite was outgassing.

The along-track force over the remainder of the mission appears to have a dependence on the angle between the earth-sun line and orbit plane which suggests that it is solar in origin. Investigations are underway to determine if it is due to radiation pressure or thermal radiation. A possible misalignment in the solar array is suspect. Perturbations in a, e, N and i will need to be examined to isolate the magnitude and direction of these forces. Once these anomalous forces are better understood, a quasi-empirical model can be developed,

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